TEMPERATURE FIELD OF ASYMMETRICALLY HEATED "THIN" AND MASSIVE TUBES WITH CONSIDERATION OF HEAT TRANSFER IN THE CLEARANCE OF THE TUBE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 4, pp. 710–715, 1968

UDC 536.12

Solutions are obtained to linearized asymmetric heat-conduction problems for thin- and thick-walled tubes, with consideration of the radiation inside the tubes. It is shown that linearization is permissible.

The heat-conduction problem involving asymmetric heating of thick-walled tubes was solved in [1] for general nonuniform boundary conditions. If the clearance of the tube is filled out with a diathermal medium, heat transfer will occur between portions of the clearance which differ in temperature. A similar phenomenon is encountered also in the design of hollow flight vehicles.

It is assumed that the degree of blackness of the wall material is unity.

Let us examine the radiation of the asymmetrically heated clearance of an infinitely long cylindrical tube (Fig. 1).

The density of the heat flux emitted by the area dF_1 onto dF_2 is equal to [2]

$$dq_{12} = \frac{\sigma T_1^4}{\pi} \frac{\cos\eta_1 \cos\eta_2 \, dF_2}{\rho^2}. \tag{1}$$

Considering the ratios between the segments and angles in a circular cylinder, instead of (1) we get

$$dq_{12} = \frac{\sigma T_1^4}{2\pi} \cos\frac{\pi - \theta + \varphi}{2} \cos^2 \varkappa \, d\theta \, d\varkappa. \tag{2}$$

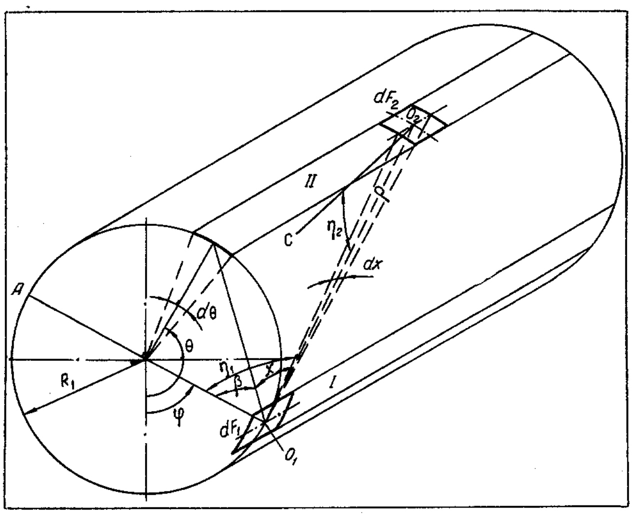

Fig. 1. Scheme for deriving an expression for the heat flux density imparted to an area element dF_1 (i.e., also to a surface unit of strip I) by the remaining portion of the clearance of an infinite tube.

The expression for the flux dq_{21} emitted by dF_2 onto the surface area dF_1 differs from the expression for dq_{12} in that T_1 is replaced by T_2. The heat flux imparted to a unit surface area on dF_1 by the entire tube clearance is obtained by integrating the difference $(dq_{21} - dq_{12})$

$$q_r = \int_{\varphi}^{2\pi+\varphi} d\theta \int_{-\pi/2}^{\pi/2} (dq_{21} - dq_{12}) \, d\varkappa =$$

$$\sigma \left[\frac{1}{4} \int_{\varphi}^{2\pi+\varphi} T^4(\theta) \sin\frac{\theta - \varphi}{2} \, d\theta - T^4(\varphi) \right]. \tag{3}$$

Let us compare (3) with the following linearized expression:

$$q_c = \frac{\bar{\alpha}}{4} \int_{\varphi}^{2\pi+\varphi} T(\theta) \sin\frac{\theta - \varphi}{2} \, d\theta - \bar{\alpha} T(\varphi), \tag{4}$$

where the mean heat-transfer coefficient in the clearance is taken as

$$\bar{\alpha} = 4\sigma T_0^3, \quad T_0 = \frac{1}{2\pi} \int_0^{2\pi} T(\varphi) \, d\varphi. \tag{4a}$$

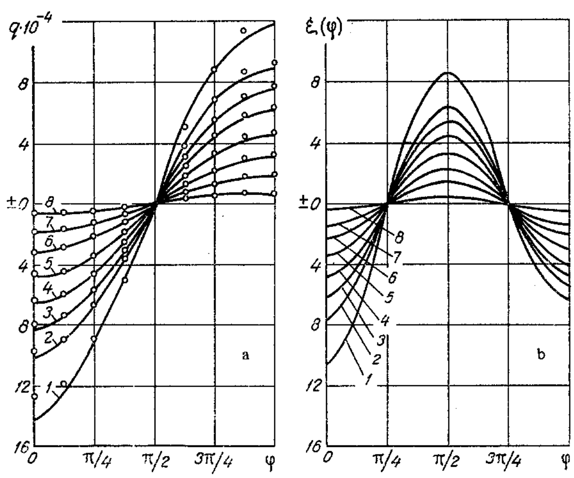

Fig. 2. Comparison of expressions (3) and (4) for $T(\varphi) = 1273 + (\Delta/2)\cos\varphi$: a) solid lines denote changes in q_r (in W/m^2) according to (3); points denote changes in q_c according to (4); b) changes in relative error (in %) $\xi(\varphi) = 10^2[q_r(\varphi) - q_c(\varphi)]/q_r(0)$ along the tube perimeter: 1) Δ = 400°K, 2) 330°K, 3) 250°K, 4) 200°K, 5) 150°K, 6) 100°K, 7) 60°K, 8) 20°K.

The comparison between (3) and (4) for $T(\varphi) = T_0 + (\Delta/2)\cos\varphi$ is given in Fig. 2. Since the maximum error occurs for $\varphi = 0$, the influence of T_0 and Δ on the value of $\xi(0)$ is shown in Fig. 3. From the figure, it can be seen that for values of Δ and T_0 frequently encountered in practice, substitution of (4) for (3) is admissible.

1. **Thin-walled tube.** Considering (4), the differential heat-conduction equation for a thin-walled tube has the form

$$\frac{\partial T(\varphi,\tau)}{\partial\tau} = \frac{a}{(R_2k)^2}\frac{\partial^2 T(\varphi,\tau)}{\partial\varphi^2} - \left[M(\tau) + \frac{\bar{\alpha}}{c\gamma\delta}\left(2 - \frac{1}{k}\right)\right]T(\varphi,\tau) + \frac{\bar{\alpha}}{4c\gamma\delta}\times \left(2 - \frac{1}{k}\right)\int_{\varphi}^{2\pi+\varphi} T(\theta,\tau)\times\sin\frac{\theta-\varphi}{2}\,d\theta + Q(\varphi,\tau). \quad (5)$$

The function $T(\varphi,\tau)$ must satisfy the condition $T(\varphi,\tau) = T(\varphi + 2\pi,\tau)$. The initial conditions are taken in the form $T(\varphi, 0) = T_i(\varphi)$. Functions $Q(\varphi,\tau)$ and $T_i(\varphi)$ must be expandable into Fourier series. The form of $Q(\varphi,\tau)$ and $M(\tau)$ depends on the type of boundary conditions at the surfaces.

If the solution to (5) is sought in the form

$$T(\varphi,\tau) = \sum_{m=0}^{\infty}[u_m^{(1)}(\tau)\cos m\varphi + u_m^{(2)}(\tau)\sin m\varphi], \quad (6)$$

where $m = 0, 1, 2, 3, \ldots$, then the expressions for $u_m^{(i)}(\tau)$ $(i = 1, 2)$ derive from ordinary differential equations which are obtained by substituting (6) into (5). Finally we get

$$u_m^{(i)}(\tau) = \frac{1}{\pi\varepsilon_m}\exp\left[-\left(\frac{am^2}{R_2^2k^2} + \bar{\alpha}\frac{2-1/k}{c\gamma\delta}\frac{4m^2}{1+4m^2}\right)\tau - \int_0^\tau M(\tau)\,d\tau\right]\left\{\int_0^\tau\int_0^{2\pi}Q(\varphi,\tau)K_m^{(i)}(\varphi)\times \exp\left[\left(\frac{am^2}{R_2^2k^2} + \bar{\alpha}\frac{2-1/k}{c\gamma\delta}\frac{4m^2}{1+4m^2}\right)\tau + \int_0^\tau M(\tau)\,d\tau\right]d\varphi\,d\tau + \int_0^{2\pi}T_i(\varphi)K_m^{(i)}(\varphi)\,d\varphi\right\}, \quad (7)$$

where $\varepsilon_m = 2$ for $m = 0$ and $\varepsilon_m = 1$ for $m = 1, 2, 3, \ldots$, $K_m^{(1)}(\varphi) = \cos m\varphi$, $K_m^{(2)}(\varphi) = \sin m\varphi$.

2. **Thick-walled tube.** The differential heat conduction equations and the initial and boundary conditions at the outer surface are taken from [1]. The boundary condition at the inner surface has the form

$$a_1\frac{\partial t(R_1,\varphi,\tau)}{\partial r} - (b_1 + \bar{\alpha})t(R_1,\varphi,\tau) = k_1\psi_1(\varphi,\tau) - \frac{\bar{\alpha}}{4}\int_{\varphi}^{2\pi+\varphi}t(R_1,\theta,\tau)\sin\frac{\theta-\varphi}{2}\,d\theta. \quad (8)$$

The solution of the problem is sought in the form

$$t(r,\varphi,\tau) = \sum_{m=0}^{\infty}[v_m^{(1)}(r,\tau)\cos m\varphi + v_m^{(2)}(r,\tau)\sin m\varphi], \quad (9)$$

where $m = 0, 1, 2, 3, \ldots$. After expansion of the functions $Q(r,\varphi,\tau)$, $\psi_1(\varphi,\tau)$, and $\psi_2(\varphi,\tau)$ into Fourier series, and having substituted (9) into the heat-conduction equation and the boundary conditions, we get a system which does not contain an integral in the boundary condition at the inner surface. Further, in order to reduce the boundary conditions to homogeneous conditions, we represent $v_m^{(i)}(r,\tau)$ $(i = 1, 2)$ in the form

$$v_m^{(i)}(r,\tau) = u_m^{(i)}(r,\tau) + z_m(r)\psi_{1,m}^{(i)}(\tau) + f_m(r)[\psi_{2,m}^{(i)}(\tau) - k_3\psi_{1,m}^{(i)}(\tau)]. \quad (10)$$

Then, for $u_m^{(i)}(r,\tau)$, we get the system

$$\frac{1}{a}\frac{\partial u_m^{(i)}}{\partial\tau} = \frac{\partial^2 u_m^{(i)}}{\partial r^2} + \frac{1}{r}\frac{\partial u_m^{(i)}}{\partial r} - \frac{m^2}{r^2}u_m^{(i)} + L(\tau)u_m^{(i)} + F_m^{(i)}(r,\tau), \quad (11)$$

$$r = R_1: \quad a_1\frac{\partial u_m^{(i)}}{\partial r} - \left(b_1 + \bar{\alpha}\frac{4m^2}{1+4m^2}\right)u_m^{(i)} = 0, \quad (12)$$

$$r = R_2: \quad a_2\frac{\partial u_m^{(i)}}{\partial r} + b_2u_m^{(i)} = 0, \quad (13)$$

$$\tau = 0: \quad u_m^{(i)}(r, 0) = y_m^{(i)}(r) - z_m(r)\psi_{1,m}^{(i)}(0) - f_m(r)[\psi_{2,m}^{(i)}(0) - k_3\psi_{1,m}^{(i)}(0)], \quad (14)$$

$$F_m^{(i)}(r,\tau) = \frac{1}{\lambda}q_m^{(i)}(r,\tau) - \frac{1}{a}\left[z_m(r)\frac{d\psi_{1,m}^{(i)}}{d\tau} + f_m(r)\left(\frac{d\psi_{2,m}^{(i)}}{d\tau} - k_3\frac{d\psi_{1,m}^{(i)}}{d\tau}\right)\right] + L(\tau)[z_m(r)\psi_{1,m}^{(i)} + f_m(r)(\psi_{2,m}^{(i)} - k_3\psi_{1,m}^{(i)})] +$$

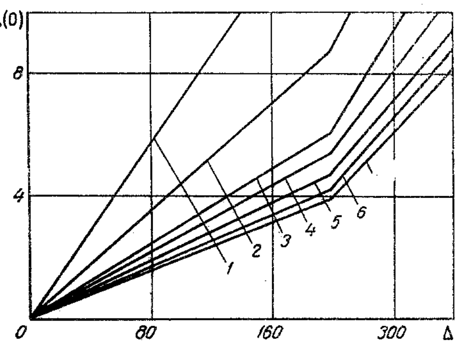

Fig. 3. Maximum error $\xi(0)$ resulting from the substitution of (3) for (4) as a function of the maximum temperature difference Δ and mean temperature T_0 of the inner tube surface for $T(\varphi) = T_0 + (\Delta/2)\cos\varphi$: 1) $T_0 = 473°$ K, 2) 773° K, 3) 1073° K, 4) 1273° K, 5) 1373° K, 6) 1473° K, 7) 1573° K.

Data Computed from (17) with Allowance ($\bar{\alpha} > 0$) and Without Allowance ($\bar{\alpha} = 0$) for the Radiation Inside the Tube Clearance

Section no. j	T_i^{av}, °K	Δ, °K	T_f^{av}, °K	kJ/kg·deg	τ_j, τ_j, sec	a, m^2/sec	$\bar{\alpha}$, W/m²·deg initial	Final	mean	$T(0,\tau_j)$, °K	$T(\pi,\tau_j)$, °K	$\Delta_K = T(0,\tau_j) - T(\pi,\tau_j)$, °K
1	273	0	573	0.52	80.5	162	4.6	43	23.8	598.6	547.4	51.2
	273	0	573	0.52	80.5	162	0.0	0.0	0.0	599.4	546.6	52.8
2	573	51.2	873	0.662	102.5	108	43	151	97	923.7	822.3	101.4
	573	52.8	873	0.662	102.5	108	0.0	0.0	0.0	931.4	814.6	116.8
3	873	101.4	1173	0.93	144	72	151	368	259.5	1238.6	1107.4	131.2
	873	116.8	1173	0.93	144	72	0.0	0.0	0.0	1275.8	1070.2	205.6

$$+\psi_{1,m}^{(i)}\left[\frac{d^2z_m}{dr^2}+\frac{1}{r}\frac{dz_m}{dr}-\frac{m^2}{r^2}z_m\right]+$$

$$+(\psi_{2,m}^{(i)}-k_3\psi_{1,m}^{(i)})\left[\frac{d^2f_m}{dr^2}+\frac{1}{r}\frac{df_m}{dr}-\frac{m^2}{r^2}f_m\right], \qquad (15)$$

$q_m^{(i)}(r,\tau)$, $\psi_{1,m}^{(i)}(\tau)$, $\psi_{2,m}^{(i)}(\tau)$, $y_m^{(i)}(r)$ are the coefficients of the Fourier series expansions in $Q(r,\varphi,\tau)$, $\psi_1(\varphi,\tau)$, $\psi_2(\varphi,\tau)$ functions and in the initial temperature distribution in the tube wall, respectively. The functions $z_m(r)$ and $f_m(r)$ must satisfy the boundary conditions (13*)–(14*)† in [1], requiring only that b_1 appearing in them be replaced by $[b_1 + \bar{\alpha}4m^2/(1+4m^2)]$. For $z_0(r)$ and $f_0(r)$, it is advantageous to take the functions obtained in [1] and for $z_m(r)$ and $f_m(r)$, the functions by means of which the terms enclosed in the last two brackets in (15) are transformed either to zero or to a constant other than zero.

Since problem (11)–(14) is analogous to (16*)–(19*) in [1], the final form of the solution will be

$$u_m^{(i)}(r,\tau)=\sum_{n=1}^{\infty}\exp\left[-\mu_{m,n}^2\frac{a\tau}{R_2^2}+\right.$$

$$\left.+a\int_0^{\tau}L(\tau)\,d\tau\right]\left\{a\int_{R_1}^{R_2}\int_0^{\tau}F_m^{(i)}(r,\tau)\times\right.$$

$$\times M_m(n,r)\exp\left[\mu_{m,n}^2\frac{a\tau}{R_2^2}-a\int_0^{\tau}L(\tau)\,d\tau\right]dr\,d\tau+$$

$$\left.+\int_{R_1}^{R_2}u_m^{(i)}(r,0)M_m(n,r)\,dr\right\}M_{m,n}(r), \qquad (16)$$

$M_m(n,r)$, $M_{m,n}(r)$, and $\mu_{m,n}$ are determined as in [1], but everywhere $[b_1 + \bar{\alpha}4m^2/(1+4m^2)]$ must be substituted for b_1. In [1], printing errors have escaped the attention of the author: $A_{m,n}$ and $B_{m,n}$ from (26*) must be divided by R_2, while the factor $(2ma_1/\mu_{m,n}^2)$ in front of the parentheses in the third line of (28*) must be additionally divided by ω. ‡

Example. It is required to determine the temperature field of a tube heated from the outside only by a heat flux $q(\varphi) = q_0 + q_1^{(1)}\cos\varphi$; the material is St. 20 steel, $R_2 = 0.075$ m, $\delta = 0.0075$ m; $q_0 = 108\ 000$, $q_1^{(1)} = 1390$ W/m², $T_i = 273°$ K. Computation will be performed by sections (averaging the thermophysical characteristics of the steel and the heat transfer coefficient $\bar{\alpha}$ over each section). Consequently, for the second and subsequent sections, we have $T_i(\varphi) = T_i^{av} + (1/2)\Delta_i\cos\varphi$. In this case, for a thin-walled tube, from (6)–(7), we have

$$T(\varphi,\tau)=T_{\text{н}}^{\text{ср}}+\frac{\Delta_{\text{н}}}{2}\cos\varphi\exp(-D\tau)+\frac{q_0\tau}{c\gamma\delta k}+$$

$$+[1-\exp(-D\tau)]\frac{q_1^{(1)}\cos\varphi}{D},$$

$$D=\frac{a}{R_2^2k^2}+\bar{\alpha}\frac{4}{5}\frac{2-1/k}{c\gamma\delta}. \qquad (17)$$

The sequence and results of computations from (17) for $\bar{\alpha} > 0$ and $\bar{\alpha} \equiv 0$ are given in a table from which it can be seen that with increasing temperature, neglect of radiation inside the tube leads to appreciable errors. This is further aggravated by the fact that $\bar{\alpha}$ increases rapidly with increasing temperature, while the thermal conductivity of the steel decreases [3].

NOTATION

T_1 and T_2 are the surface temperature of the areas dF_1 and dF_2, respectively; ρ is the spacing between the centers of the areas dF_1 and dF_2; R_1 and R_2 are the inner and outer radii of the tube, respectively; T and t are the instantaneous temperatures of a thin- and thick-walled tube, respectively; r, φ, and τ are the instantaneous radius, angle, and time; σ is the radiation constant of an absolute blackbody ($\sigma \approx 5.7$ W/m²/°K⁴); $\bar{\alpha}$ is the mean heat-transfer coefficient of the entire tube clearance and the strip with the angular coordinate φ; a and λ are the thermal-diffusivity and thermal-conductivity coefficients; c and γ are the mean specific heat and specific weight of the tube material; $\delta = (R_2 - R_1)$ is the wall thickness of the tube; $k = 1 - \delta/2R_2$; $\omega = R_1/R_2$.

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† Here, and in the following, the asterisk denotes the number of the formulas in [1].

‡ The expression under the summation sign in (34*) must be multiplied by $M_{0,n}(r)$.

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5 February 1968

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